

[This question paper contains 2 printed pages.]

Serial No. of Question Paper : 8897

(14)

Your Roll No. 2019

Unique Paper Code : 235104

~~Paper Code : MAHT-103~~

Name of the Course : B.Sc. (Hons.) Mathematics

Name of the Paper : Algebra

Semester : I

Duration: 3 Hours

Maximum Marks: 75

Instruction for Candidates

- 1) All six questions are compulsory.
- 2) Do any two parts from each question.
- 3) Marks for each part of a question are written against the question in the margin.

1. a) Find the polar representation of complex number

$$z = \cos a - i \sin a, a \in [0, 2\pi)$$

- b) Compute the following

$$z^n + \frac{1}{z^n} \text{ if } z + \frac{1}{z} = \sqrt{3}.$$

- c) Find the quadratic equation whose roots are the cubes of the roots of the equation $x^2 - px + q = 0$.

2. a) For $a, b \in \mathbb{R} \setminus \{0\}$, define $a \sim b$ if and only if $\frac{a}{b} \in \mathbb{Q}$

i. Prove that \sim defines an equivalence relation.

ii. What is an equivalence class of 1? Show that $\sqrt{3} = \sqrt{12}$.

- b) Given three consecutive integers $a, a + 1, a + 2$, prove that one of them is divisible by 3.

- c) a) Define $f: \mathbb{Z} \rightarrow \mathbb{Z}$ by

$$f(x) = 3x^3 + x.$$

Determine whether or not f is one to one and/or onto.

3. a) Use mathematical induction to establish that for all $n \geq 1$, $8^n - 3^n$ is divisible by 3.

- b) Show that the set of rational numbers is countable.

c) Find all integers x , $0 \leq x < 6$, satisfying the following congruence
 $4x \equiv 2 \pmod{6}$.

a) Find the general solution of the system

$$2x_1 - x_2 + x_3 + 2x_4 = 0$$

$$-2x_1 + 4x_2 - x_3 + 2x_4 = -5$$

$$x_1 - 6x_2 + 3x_3 + x_4 = 7$$

$$4x_1 - 6x_2 + x_3 - 4x_4 = 9$$

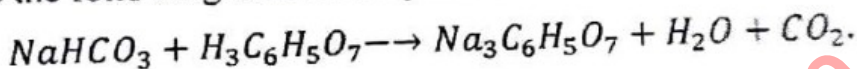


by reducing the coefficient matrix to echelon form.

b) Determine whether b belongs to the linear span of a_1 , a_2 and a_3 , where

$$a_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, a_2 = \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix}, a_3 = \begin{pmatrix} -6 \\ 7 \\ 5 \end{pmatrix}, \text{ and } b = \begin{pmatrix} 11 \\ -5 \\ 9 \end{pmatrix}.$$

c) Balance the following chemical equation



5.

a) For what values of h the vectors v_1 , v_2 and v_3 given below

$$v_1 = \begin{pmatrix} 3 \\ -6 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} -6 \\ 4 \\ -3 \end{pmatrix}, v_3 = \begin{pmatrix} 9 \\ h \\ 3 \end{pmatrix},$$

are linearly dependent?

b) Let $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $y_1 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, and $y_2 = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$. Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be such that $Te_1 = y_1$ and $Te_2 = y_2$. Find $T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

c) (i) Let $T: \mathbb{R}^n \rightarrow \mathbb{R}^n$ be a linear transformation. Show that T is one-to-one if and only if $T(x) = 0$ has only the trivial solution.
 (ii) Show that $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by $T(x_1, x_2) = (x_1 + x_2, x_2)$ is one-to-one.

6.

a) Find the standard matrix of the horizontal sheer transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ leaves e_1 unchanged and maps e_2 into $e_2 + 2e_1$.

b) Let $T: \mathbb{R}^n \rightarrow \mathbb{R}^n$ be a linear transform and A be standard matrix representation of T . Show that T is invertible linear transformation if and only if A is an invertible matrix.

c) Determine the rank of the matrix

$$\begin{pmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & 2 & 4 \\ 0 & -9 & 6 & 5 & -6 \end{pmatrix}.$$